Determine if each of the following functions is continuous. **STATE YOUR CONCLUSIONS CLEARLY.** SCORE: _____ / 6 PTS
If a function is continuous, justify your conclusion using the definition(s) and/or theorems.
If a function is not continuous, show clearly which part of the definition of "continuous" is not true.

[a] $f(x)=\begin{cases} x^4-x^3-1, & \text{if } x\le 2 \\ x^5-10x-5, & \text{if } x>2 \end{cases}$

1 $\boxed{f \text{ IS CONT. AT } x\neq 2 \text{ (POLYNOMIAL)}}$

$f(2)=2^4-2^3-1=7$

$\frac{1}{2}$ $\boxed{\lim_{x\to 2^+}(x^5-10x-5)}=2^5-10(2)-5=7$

$\frac{1}{2}$ $\boxed{\lim_{x\to 2^-}(x^4-x^3-1)}=2^4-2^3-1=7$

1 $\boxed{\lim_{x\to 2} f(x)=7}=\boxed{f(2)}$ 1 $\rightarrow f$ IS CONT AT $x=2 \rightarrow$ $\boxed{f \text{ IS CONT}}$ 1

[b] $f(x)=\begin{cases} \frac{x^3+1}{x^2-1}, & \text{if } x<-1 \\ \frac{x^2-4}{x+3}, & \text{if } x>-1 \end{cases}$

$\boxed{\begin{array}{l} f(-1) \text{ DNE} \\ f \text{ NOT CONT} \end{array}}$ 1

Let $f(x)=\sqrt{29-4x}$. SCORE: _____ / 8 PTS

[a] Find $f'(x)$.

1 $\boxed{\lim_{h\to 0}\frac{\sqrt{29-4(x+h)}-\sqrt{29-4x}}{h}}\cdot\frac{\sqrt{29-4(x+h)}+\sqrt{29-4x}}{\sqrt{29-4(x+h)}+\sqrt{29-4x}}$

$=\lim_{h\to 0}\frac{29-4(x+h)-(29-4x)}{h(\sqrt{29-4(x+h)}+\sqrt{29-4x})}$

2 $=\boxed{\lim_{h\to 0}\frac{-4\cancel{h}}{\cancel{h}(\sqrt{29-4(x+h)}+\sqrt{29-4x})}}=\frac{-4}{2\sqrt{29-4x}}=\boxed{\frac{-2}{\sqrt{29-4x}}}$ 1

[b] Find the slope-point form of the equation of the tangent line to the curve of $f(x)$ at the point where $x=1$.

$f'(1)=\frac{-2}{\sqrt{25}}=\boxed{-\frac{2}{5}}$ 1 $\boxed{y-5=-\frac{2}{5}(x-1)}$ 1

[c] The position (in yards) of an object moving in a straight line is given by $s(t)=\sqrt{29-4t}$, where t is the time in minutes.
Find the instantaneous velocity of the object at time $t=5$. Give the correct units for your answer.

$s'(5)=\frac{-2}{\sqrt{9}}=\boxed{-\frac{2}{3}}$ 1 $\boxed{\text{YARD/MINUTE}}$ 1

Using complete sentences and proper mathematical notation, write the formal definition of "derivative (function)". SCORE: _____ / 1 PT

THE DERIVATIVE OF f IS $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

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Using complete sentences and proper mathematical notation, write the formal definition of "continuous (at a point)". **SCORE: _____ / 2 PTS**

f IS CONTINUOUS AT a IFF $f(a)$ EXISTS, $\lim_{x\to a} f(x)$ EXISTS AND $\lim_{x\to a} f(x) = f(a)$

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The time it takes to recover from a certain illness depends on the daily dosage of a certain medicine. **SCORE: _____ / 2 PTS**
Suppose $r = f(d)$, where r is the recovery time (in days), and d is the daily dosage (in grams)

[a] What does $f(10) = 6$ mean ? Give the correct units for all numbers in your answer.

IT TAKES 6 DAYS TO RECOVER IF THE DAILY DOSAGE IS 10g.

[b] What does $f'(10) = -0.5$ mean ? Give the correct units for all numbers in your answer.

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IF THE DAILY DOSAGE IS 10g, YOU WILL RECOVER $\approx \frac{1}{2}$ DAY SOONER FOR EACH ADDITIONAL 1g YOU TAKE EACH DAY.

Find the following limits. **SCORE: _____ / 7 PTS**
Each answer should be a number, ∞, $-\infty$, or DNE (only if the other answers do not apply).

[a] $\lim_{x\to -\infty} \tan^{-1} x$

$= \boxed{-\frac{\pi}{2}}$ 1

[b] $\lim_{x\to -\infty} \frac{\sqrt{36x^2 - 49x}}{4 - 5x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to -\infty} \frac{\sqrt{36x^2-49x} \cdot -\sqrt{\frac{1}{x^2}}}{\frac{4}{x} - 5}$ 1

$= \lim_{x\to -\infty} \boxed{\frac{-\sqrt{36 - \frac{49}{x}}}{\frac{4}{x} - 5}}$ 2

$= \frac{-\sqrt{36-0}}{0-5} = \boxed{\frac{6}{5}}$ 1

[c] $\lim_{x\to \infty} \arccos e^{-x}$

$= \boxed{\arccos 0}$ 1

$= \boxed{\frac{\pi}{2}}$ 1

Prove that $\tan x = \cos x$ for some x in the interval $(0, \frac{\pi}{4})$. **DO NOT ATTEMPT TO SOLVE FOR x.** **SCORE: _____ / 4 PTS**

1 LET $f(x) = \tan x - \cos x$. f IS CONTINUOUS ON $[0, \frac{\pi}{4}]$ SINCE IT IS THE DIFFERENCE OF CONTINUOUS FUNCTIONS.

$f(0) = \tan 0 - \cos 0 = -1$ AND $f(\frac{\pi}{4}) = \tan\frac{\pi}{4} - \cos\frac{\pi}{4} = 1 - \frac{\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$

$\boxed{-1 < 0 < \frac{2-\sqrt{2}}{2}}$, SO $\boxed{\text{BY IVT}}$, FOR SOME $c \in (0, \frac{\pi}{4})$, $f(c) = 0$, IE. $\tan c - \cos c = 0$ OR $\tan c = \cos c$ 1

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